Read the statements A and B given below in the view of non-contemporaneous variation of a functional, $\underset{y(x)}{\text{Min}}\ J = \int_{x_1}^{x_2} F(y(x), y'(x), y''(x))dx$, and answer questions from 1- 4.

A. The Euler-Lagrange equation is

$$F_y - (F_{y'})' + (F_{y''})'' = 0 \quad (1.1)$$

B. Boundary conditions are

$$\left(F_{y''}\delta y'\right)\Big|_{x_1}^{x_2} = 0 \quad (1.2)$$

$$\left\{\left(F_{y'} - \left(F_{y''}\right)'\right)\delta y\right\}\Big|_{x_1}^{x_2} = 0 \quad (1.3)$$

$$\left\{\left(F - F_{y'}y' + \left(F_{y''}\right)' y' - F_{y''}y''\right)\delta x\right\}\Big|_{x_1}^{x_2} = 0 \quad (1.4)$$

1. Derive the Euler-Lagrange equation and boundary conditions for general variations starting from the minimization statement and compare your solution with statements A and B, and then select the appropriate option given below.

a) Statement A is correct but not B.
b) Statement B is correct but not A.
c) Both statements A and B are correct.
d) Both statements A and B are wrong.

2. While deriving the necessary conditions in Question 1, you would have accounted for *h*, the difference between the perturbed and original curves at the end points. Which of the following captures the end-point variations?

a) $\delta y_1 = h_1 + y_1'\delta x_1$ and $\delta y_2 = h_2 + y_2'\delta x_2$
b) $\delta y_1 = h_1 = y_1'\delta x_1$ and $\delta y_2 = h_2 = y_2'\delta x_2$
c) $\delta y_1 = h_1 - y_1'\delta x_1$ and $\delta y_2 = h_2 + y_2'\delta x_2$
d) $\delta y_1 = h_1 + y_1'\delta x_1$ and $\delta y_2 = h_2 - y_2'\delta x_2$

3. Boundary conditions for the minimizing statement, $\underset{y(x)}{\text{Min}}\ J = \int_{x_1}^{x_2} F(y(x), y'(x))dx$, can be obtained from Statement B by setting … (Note : Use correct statements A and B from Question 1)

a) $\delta y = 0$ and $\delta y' = 0$
b) $\delta y'' = 0$ and $y'' = 0$
c) $F_{y'} = 0$ and $\delta y' = 0$
d) $F_{y''} = 0$ and $y'' = 0$

4. Which of the following statement is false?

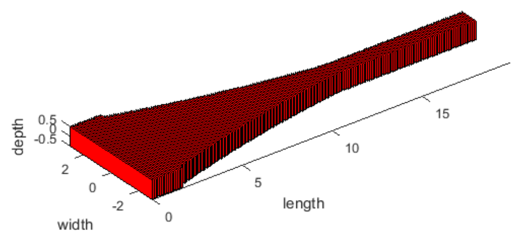
a) Euler-Lagrange equations does not change for non-contemporaneous variations.
b) The only change in necessary conditions for a non-contemporaneous variation is Equation 1.4 given at the beginning.
c) Transversality conditions for an optimal solution with end points constrained on two curves can be obtained from Statement 2.
d) The guided-beam problem cannot be solved using Statements A and B.

5. Earlier in the course, we had derived Snell's law using finite-variable optimization. Which of the following conditions is required if we want to derive Snell's law using calculus of variations?

a) Noether's theorem
b) Transversality conditions
c) Complimentarity conditions
d) Weierstrass-Erdmann corner conditions

Study the Matlab code provided, beamOpt.m, and answer questions from 6-10.

6. Given below is a final area profile from optimization code for a stiffest beam under uniform loading. Can you guess the boundary conditions of the beam?



a) Fixed-free
b) Fixed-fixed
c) Fixed-guided
d) None of the above

7. Which of the following Matlab statements checks for area and set it to the upper limit of the area.

a)
```
if (A(j)>Amax)
    A(j)=Amax;
end
```
b)
```
if A(j)~=Amax && A(j)~=Amin
    A(j)=A(j)*(alpha*E(j)*uDashDash(j)^2/lambda)^eta;
end
```
c)
```
if (A(j)<Amin)
    A(j)=Amin;
End
```
d)
```
updatePlot(A,d,nx,n,Amax,Amin);
```

8. If you were to modify BarOpt2.m to a code equivalent to beamOpt.m, then which of the following changes are necessary?

A. feambeam() has to be used instead of feambar().
B. All the variables needs to be renamed.
C. Both first and second derivatives of displacement have to obtained using finite-difference instead of just the first derivative.
D. Optimality criterion for a stiff-beam is different from that of a stiff-bar, so the Matlab statement to update area has to be changed accordingly.
E. Inner loop that checks for upper and lower bounds on areas has to be modified.

a) A,B,C,D,E
b) A,C,D
c) A,B,C,D
d) A,C,E

9. Which of the following Matlab statements update area using the optimality criterion?

a) lambda1=lambda1+A(j)*l(j)*(alpha*E(j)*uDashDash(j)^2)^eta;
b)
```
if (A(j)>Amax)
        A(j)=Amax;
end
```
c) A(j)=A(j)*(alpha*E(j)*uDashDash(j)^2/lambda)^eta
d) updatePlot(A,d,nx,n,Amax,Amin)

10. Find the number of outer-loop iterations beamOpt.m takes to converge when eta=.2, tol = 1e-3, n = 200 for fixed-free boundary conditions. (Note: Do not change anything other than the specified in the provided code to get the correct answer)

a) 50
b) 34
c) 19
d) 4